Cities and Galaxies

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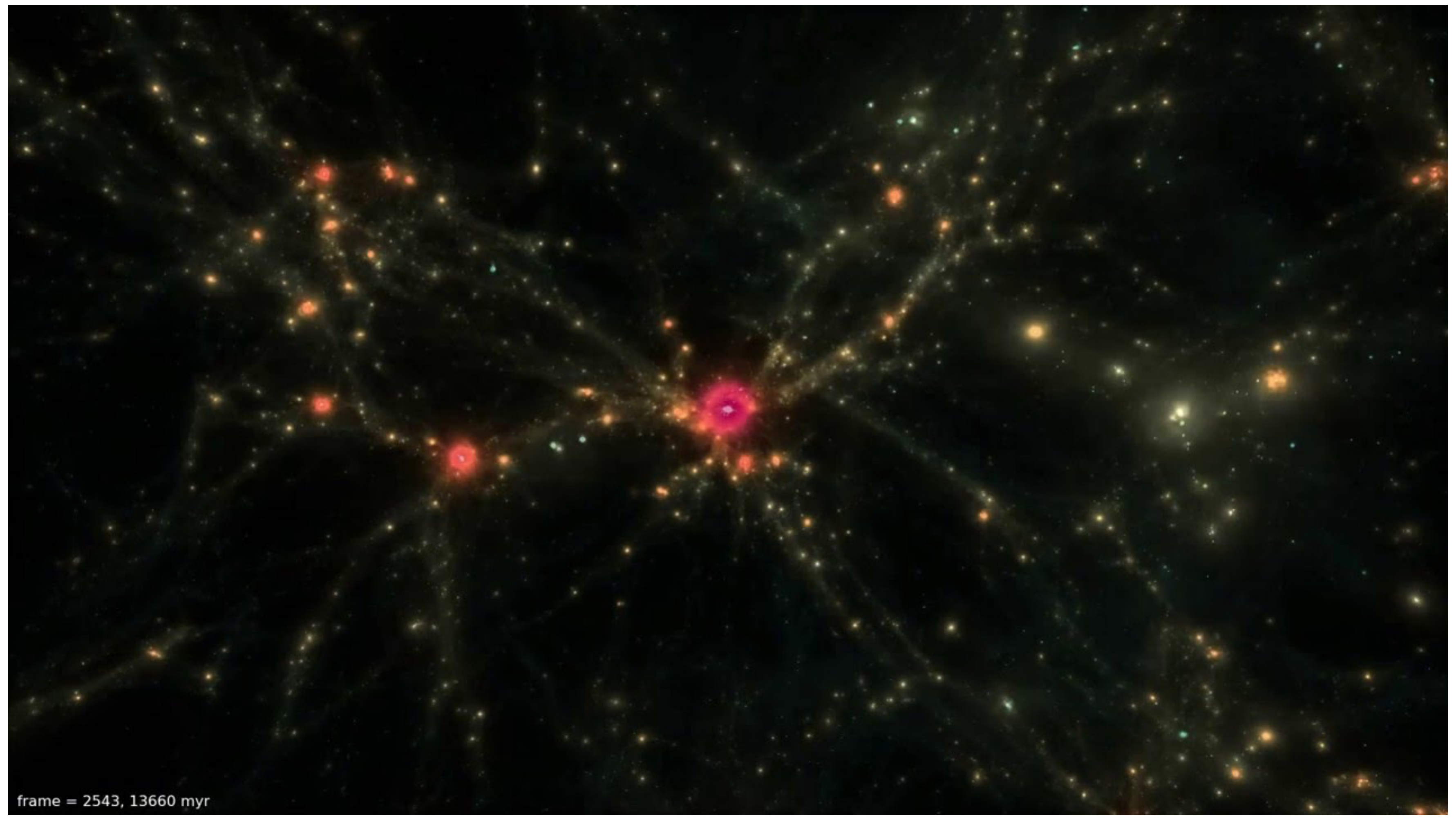
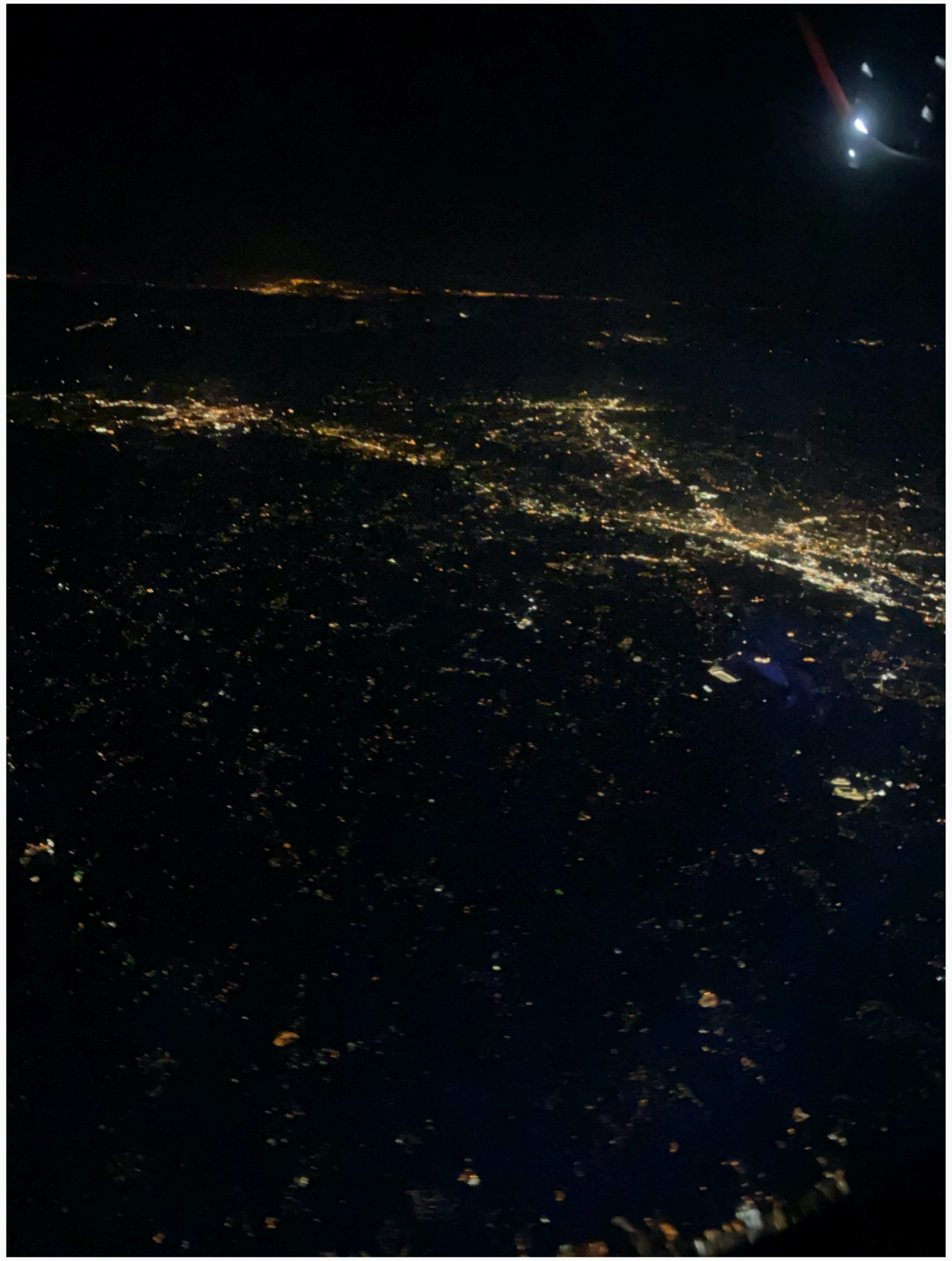
Motivation

- Was on a late night flight in February, noticed the view outside looks like large scale structure.
- Looked it up as soon as I landed.



Large Scale Structure



This looks like large scale structure.

The growth of human populations looks like the evolution of the universe.

Can be used as a topic for emergence course presentation.

Zipf's Law from Scale-free Geometry

Henry W. Lin[1] and Abraham Loeb[2]

[1] *Harvard College, Cambridge, MA 02138, USA*

[2] *Institute for Theory & Computation, Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA*

(Dated: February 16, 2016)

The spatial distribution of people exhibits clustering across a wide range of scales, from household ($\sim 10^{-2}$ km) to continental ($\sim 10^{4}$ km) scales. Empirical data indicates simple power-law scalings for the size distribution of cities (known as Zipf's law) and the population density fluctuations as a function of scale. Using techniques from random field theory and statistical physics, we show that these power laws are fundamentally a consequence of the scale-free spatial clustering of human populations and the fact that humans inhabit a two-dimensional surface. In this sense, the symmetries of scale invariance in two spatial dimensions are intimately connected to urban sociology. We test our theory by empirically measuring the power spectrum of population density fluctuations and show that the logarithmic slope $\alpha = 2.04 \pm 0.09$, in excellent agreement with our theoretical prediction $\alpha = 2$. The model enables the analytic computation of many new predictions by importing the mathematical formalism of random fields.


Cosmology

Two Point Autocorrelation Function

- "Given a random galaxy in a location, the correlation function describes the probability that another galaxy will be found within a given distance."[1]

- We use overdensity $\delta(\mathbf{x}) \equiv [(\rho(\mathbf{x})/\bar{\rho}) - 1]$

- The two-point correlation function is defined as:
$$\xi\left(\left|\mathbf{x}_1 - \mathbf{x}_2\right|\right) = \left\langle \delta\left(\mathbf{x}_1\right) \delta\left(\mathbf{x}_2\right) \right\rangle$$

$$= \frac{1}{V} \int d^3\mathbf{x} \delta(\mathbf{x}) \delta(\mathbf{x} + \mathbf{r}), \text{ where } \mathbf{r} = \left|\mathbf{x}_1 - \mathbf{x}_2\right|$$

Matter Power Spectrum

- In Fourier space,

$$\xi(r) = \int \frac{d^3k}{(2\pi)^3} \delta_{\mathbf{k}} e^{i\mathbf{k}\cdot(\mathbf{x_1} - \mathbf{x_2})}$$

- Power spectrum is defined as:

$\left\langle \delta_{\mathbf{k}} \delta'_{\mathbf{k}} \right\rangle = (2\pi)^3 \delta_D^3 \left(\mathbf{k} - \mathbf{k}'\right) P(k)$, where δ_D is the Dirac-delta function.

Matter Power Spectrum

- Conventional to define dimensionless power spectrum $\Delta^2(k) = \dfrac{k^3 P(k)}{2\pi^2}$

Matter Power Spectrum

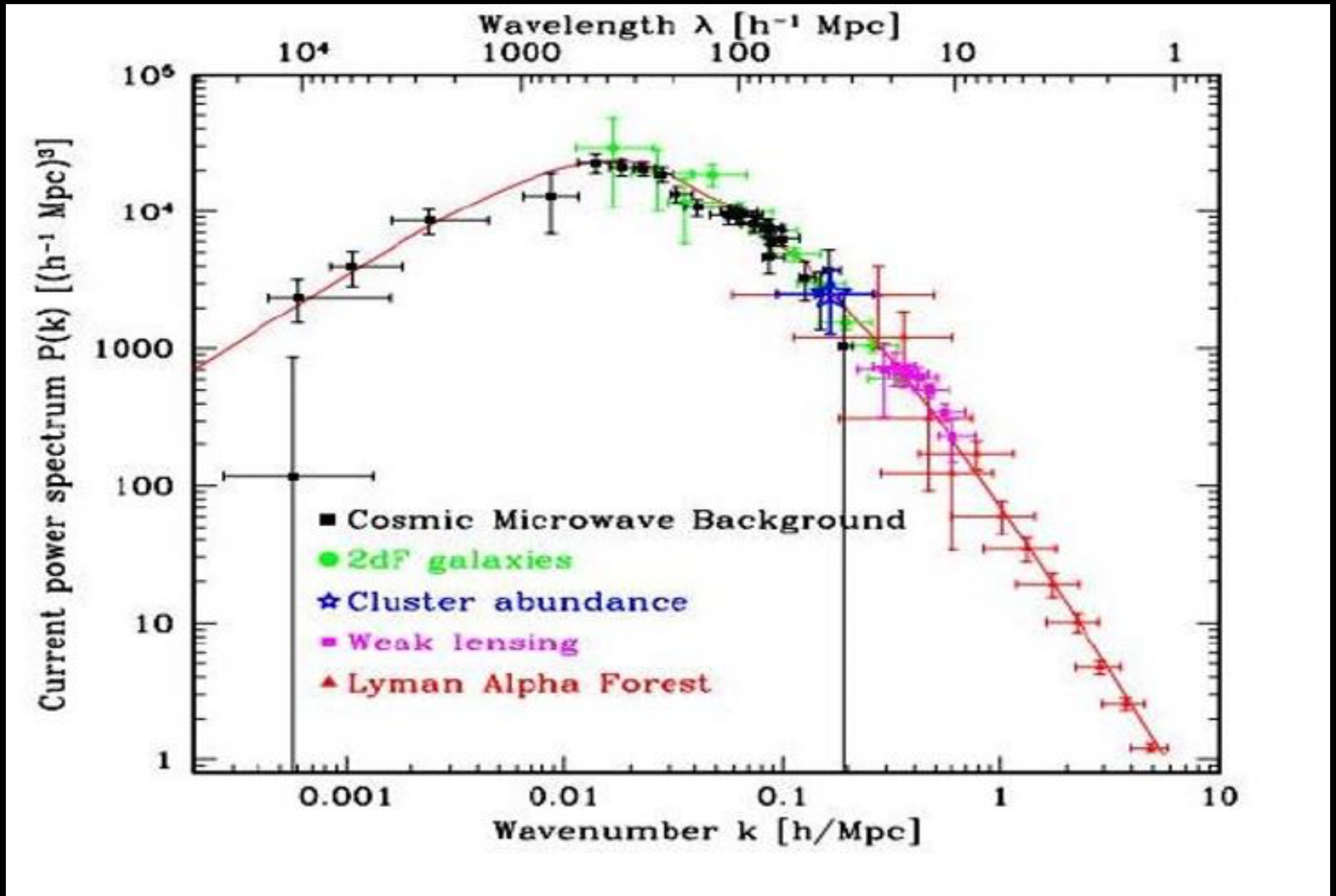


Fig. 1 Observed Power Spectrum[2]

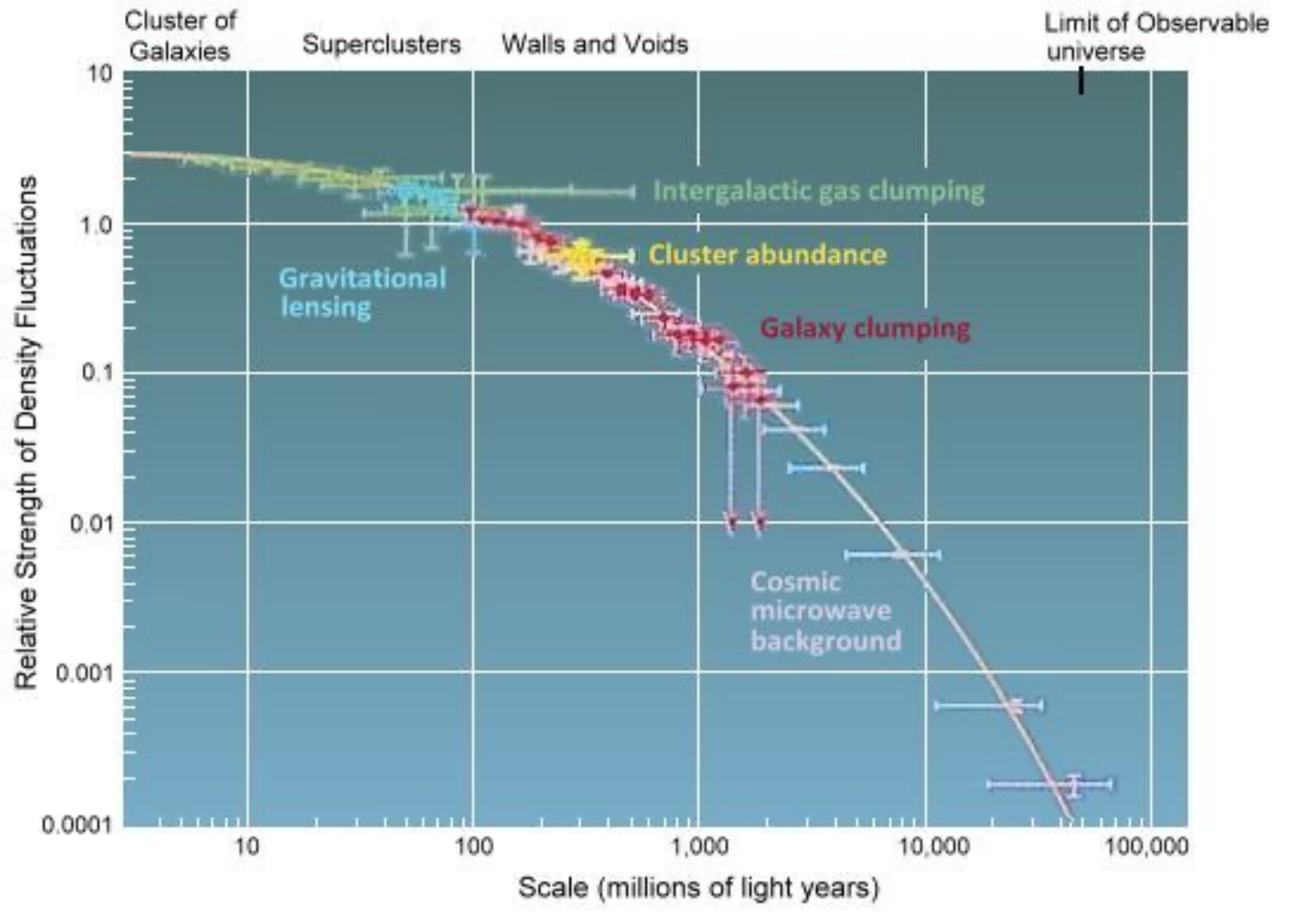


Fig. 2 Observed Power Spectrum[3]

Zipf’s Law for Cities

Zipf's Law

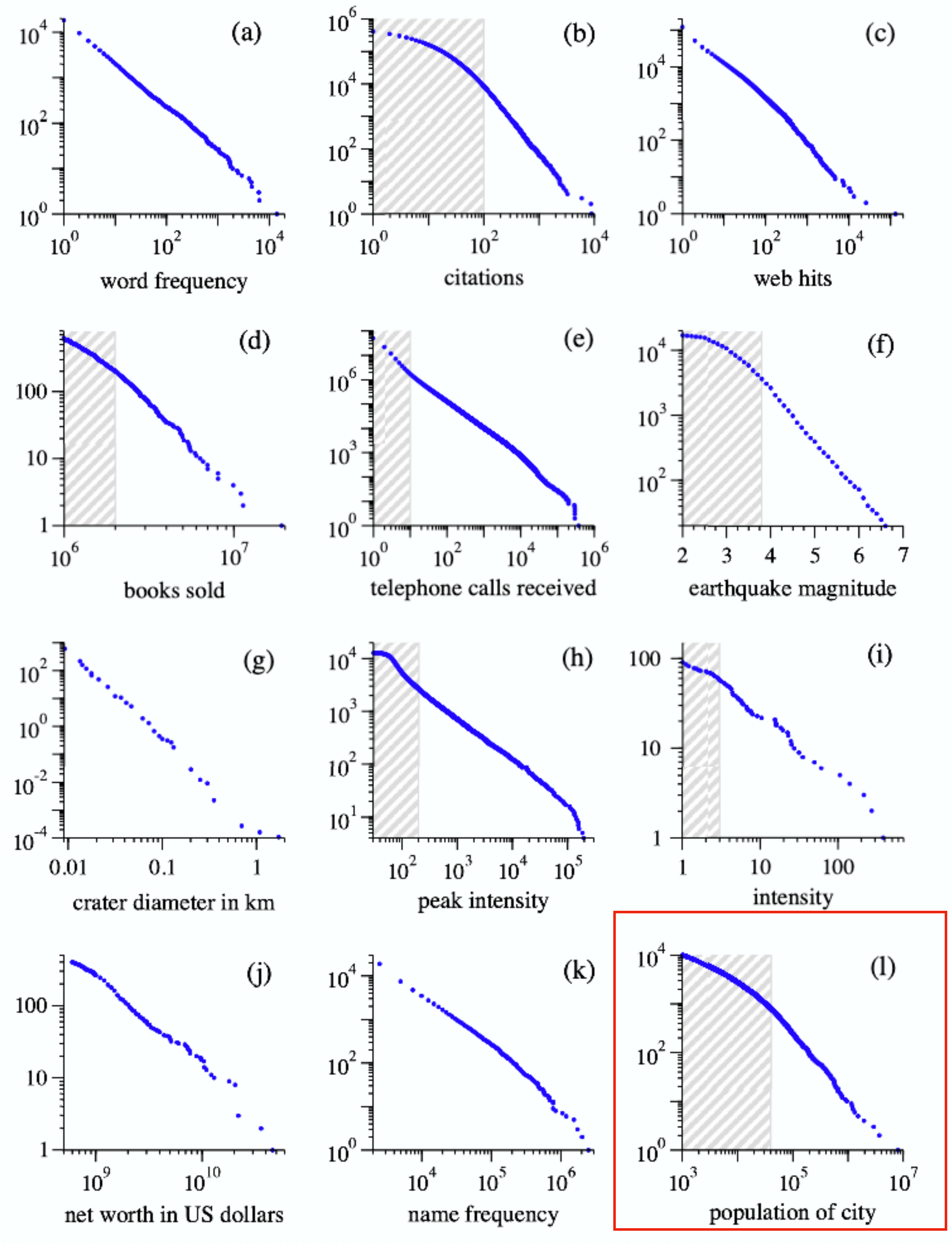


Fig 3. Rank/ Frequency Plots (Newman's paper) [4]

Zipf's Law

- The rank of a city is inversely proportional to the number of people who live there.
- $P(N) \propto \dfrac{1}{N^2}$

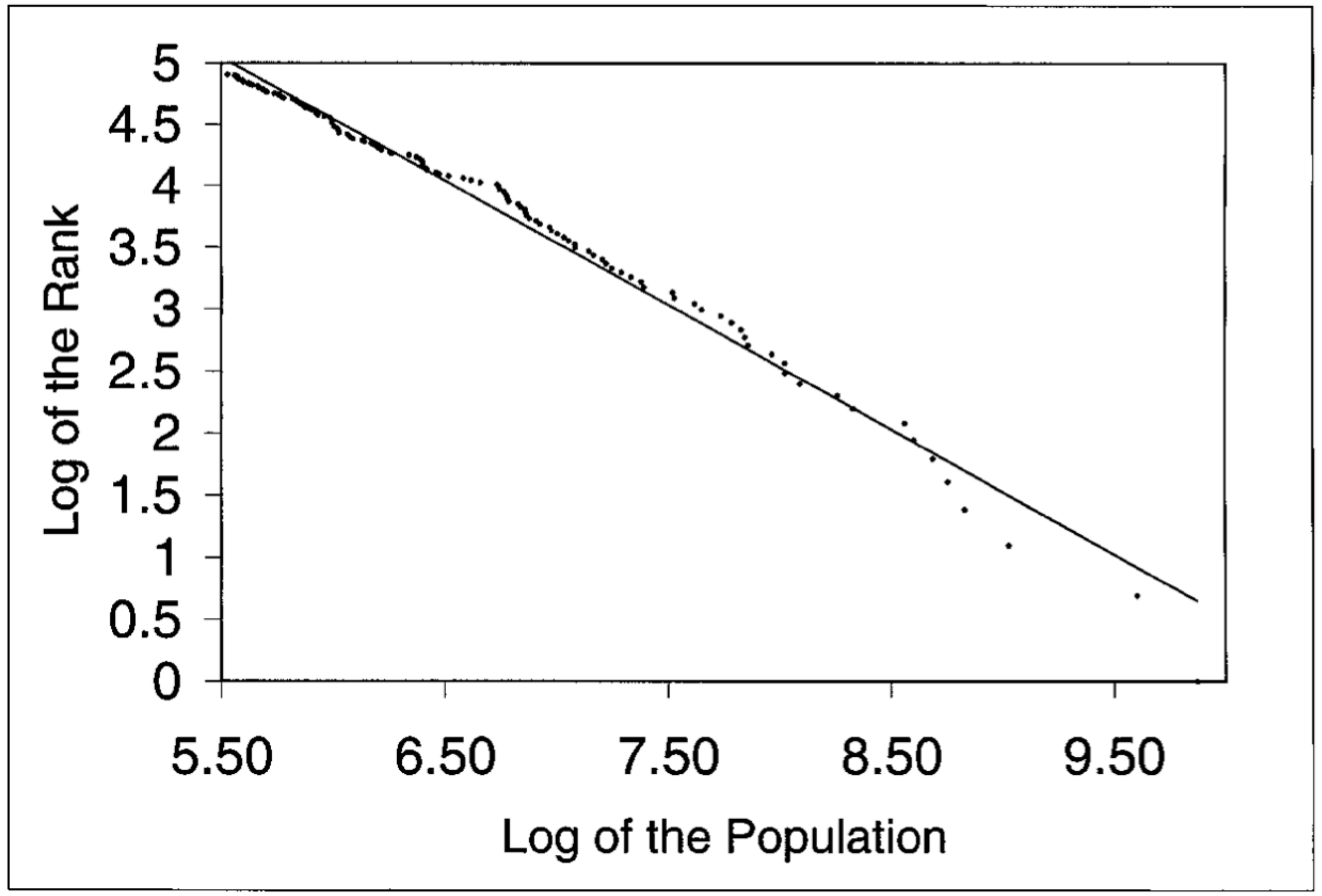


Fig 4. Log-log plot of Size vs Rank for 135 largest US metropolitan areas in 1991[5]

Setup

- Population density ρ over plane $\mathbb{R}^2$
- We study the population density fluctuation, $\delta(\mathbf{x}) \equiv [(\rho(\mathbf{x})/\bar{\rho}) - 1]$ where $\bar{\rho}$ is the average density

Setup

Fourier expansion of population fluctuation:

$$\delta(\mathbf{x}) = \frac{1}{2\pi} \int d^2k \delta_{\mathbf{k}} e^{-i\mathbf{k}\mathbf{x}}$$

Setup

- Power Spectrum in 2D: $\left\langle \delta_{\mathbf{k}} \delta_{\mathbf{k}'} \right\rangle = (2\pi)^2 \delta_D^2 (\mathbf{k} - \mathbf{k}') \, P(k)$
- Dimensionless: $\Delta^2(k) = \dfrac{k^2 P(k)}{(2\pi)}$, represents $(\dfrac{\delta\rho}{\rho})^2$ over scale $\dfrac{1}{k}$

Getting to Zipf's Law

- Consider an overdensity of size $\dfrac{1}{k}$
- The habitat can expand or contract at each time step.
- Spatial extent changes, but overdensity remains constant.

Getting to Zipf's Law

- Define a monotonically decreasing function $X(k)$
- Measure of spatial extent of an overdensity (eg $X(k) \propto 1/k$ or $1/k^2$)
- $\lim_{k \to \infty} X = 0$
- For an infinite landmass, overdensity tends to 0.

Getting to Zipf's Law

- Change of variables: $\Delta(X(k)) = \Delta(k)$
- Random walk in X,
- Till overdensity disappears or reaches some maximum X_{max}
- For a continental length scale $1/k_{min}$, $X_{max} = X(kmin)$

Getting to Zipf’s Law

- For a large ensemble of overdensities, this leads to a diffusion-like process

$$\frac{\partial \Delta}{\partial t} = D \frac{\partial^2 \Delta}{\partial X^2}$$

- For a long enough timescale, this will settle to a stead-state solution $\Delta(X) \rightarrow Constant$, for $T_{relax} \sim \frac{X^2}{D}$

- We went over this in class for the Casino earnings problem (1D Diffusion)

Getting to Zipf's Law

- Under these conditions, we get

  $$P(k) \propto k^{-2}$$

- Using this power spectrum $P(k)$, we can calculate populations for different areas.

- A city is defined as an area A where the density of population (and overdensity) is greater than some threshold δ_c

  $$N = \int_{x \in A} \rho(\mathbf{x}) d^2x = \rho_C \times A$$

Experimental Confirmation

- Empircally measured:

  $P(k) \propto k^{-\alpha},$

  where $\alpha = -2.04 \pm 0.09$

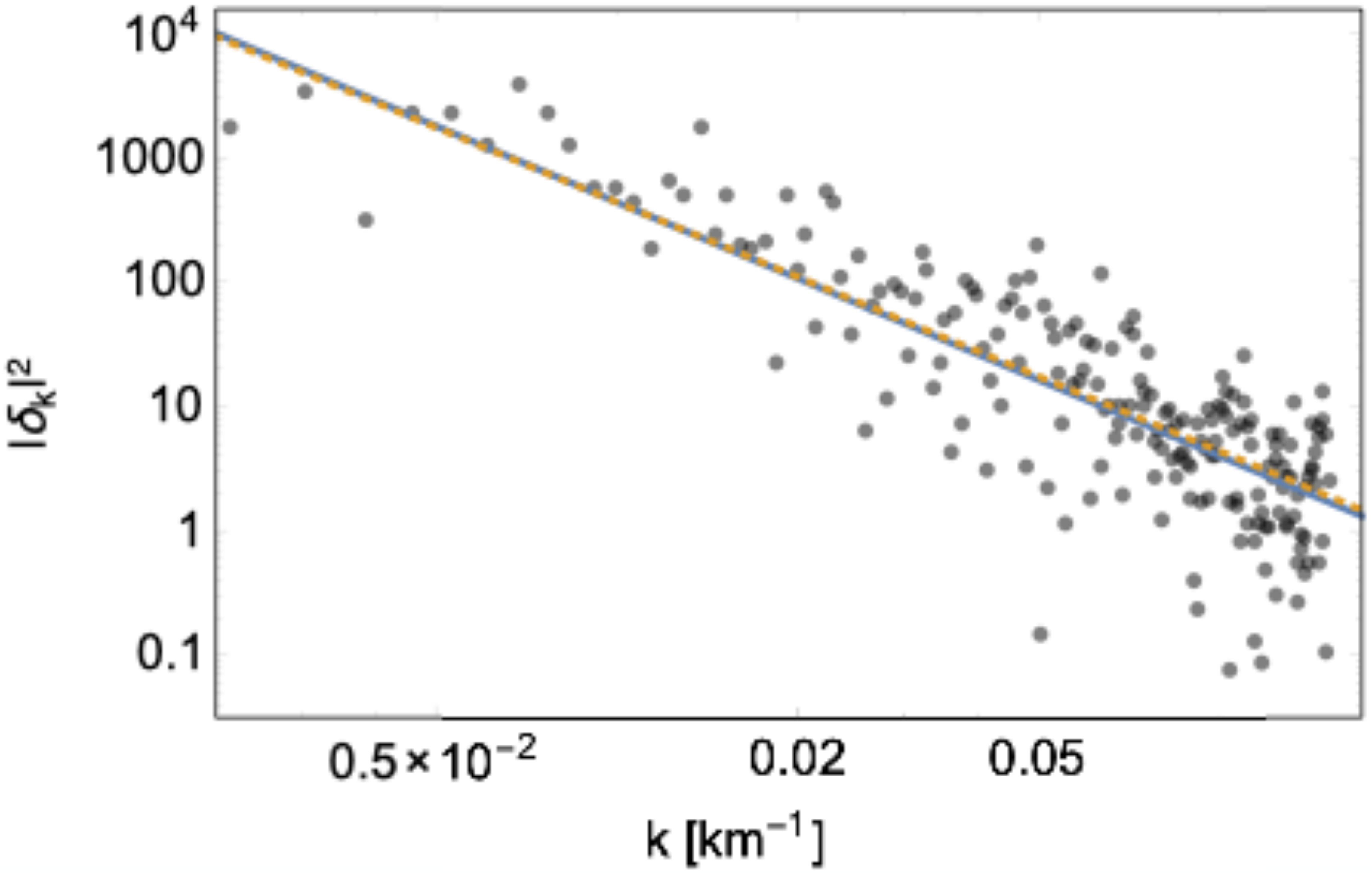


Fig. 6 Empirically measured power spectrum vs predicted

Computational Simulation

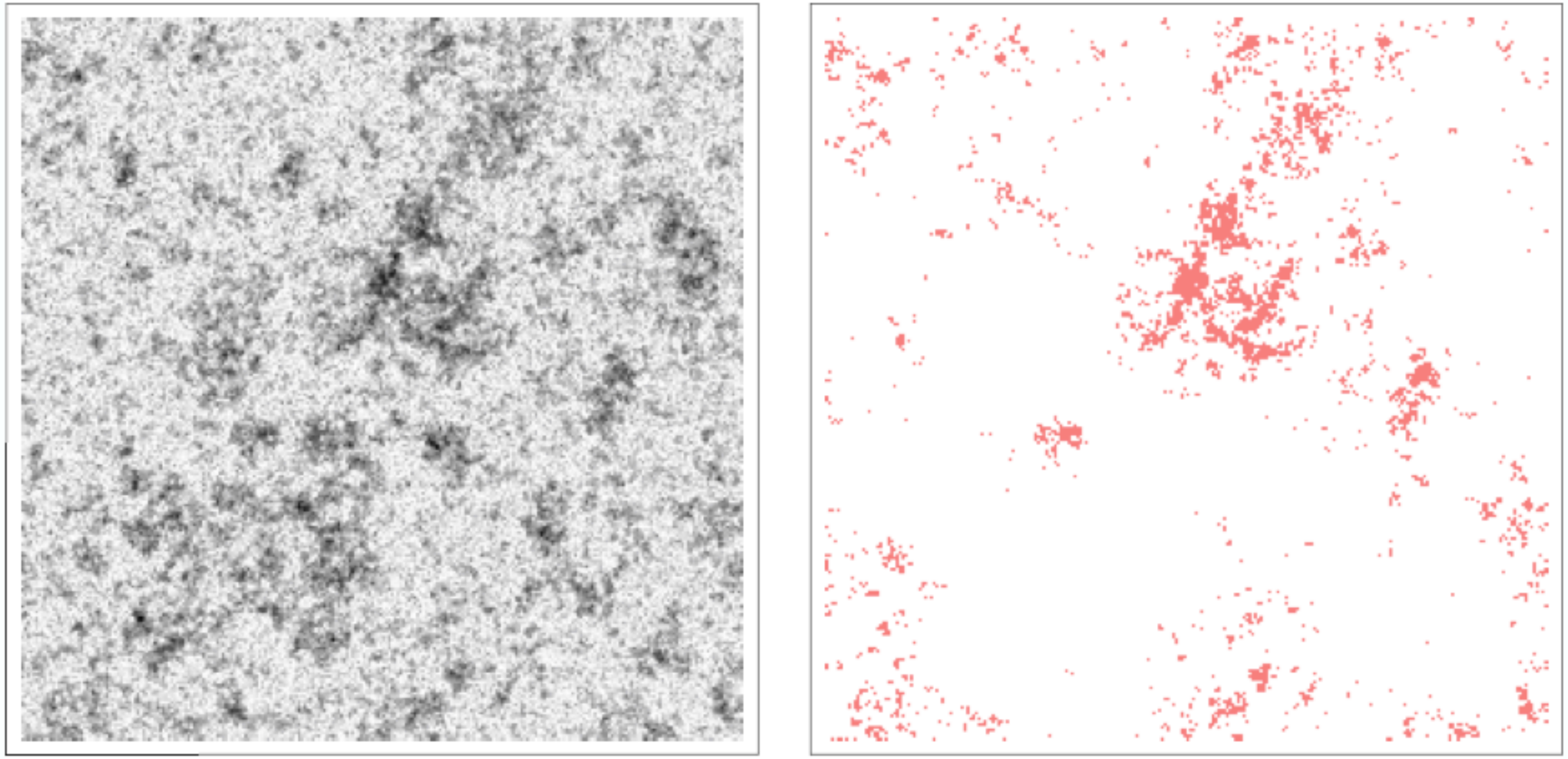


Fig. 7 Monte Carlo simulation of population density distribution, each connected component is a city

Field

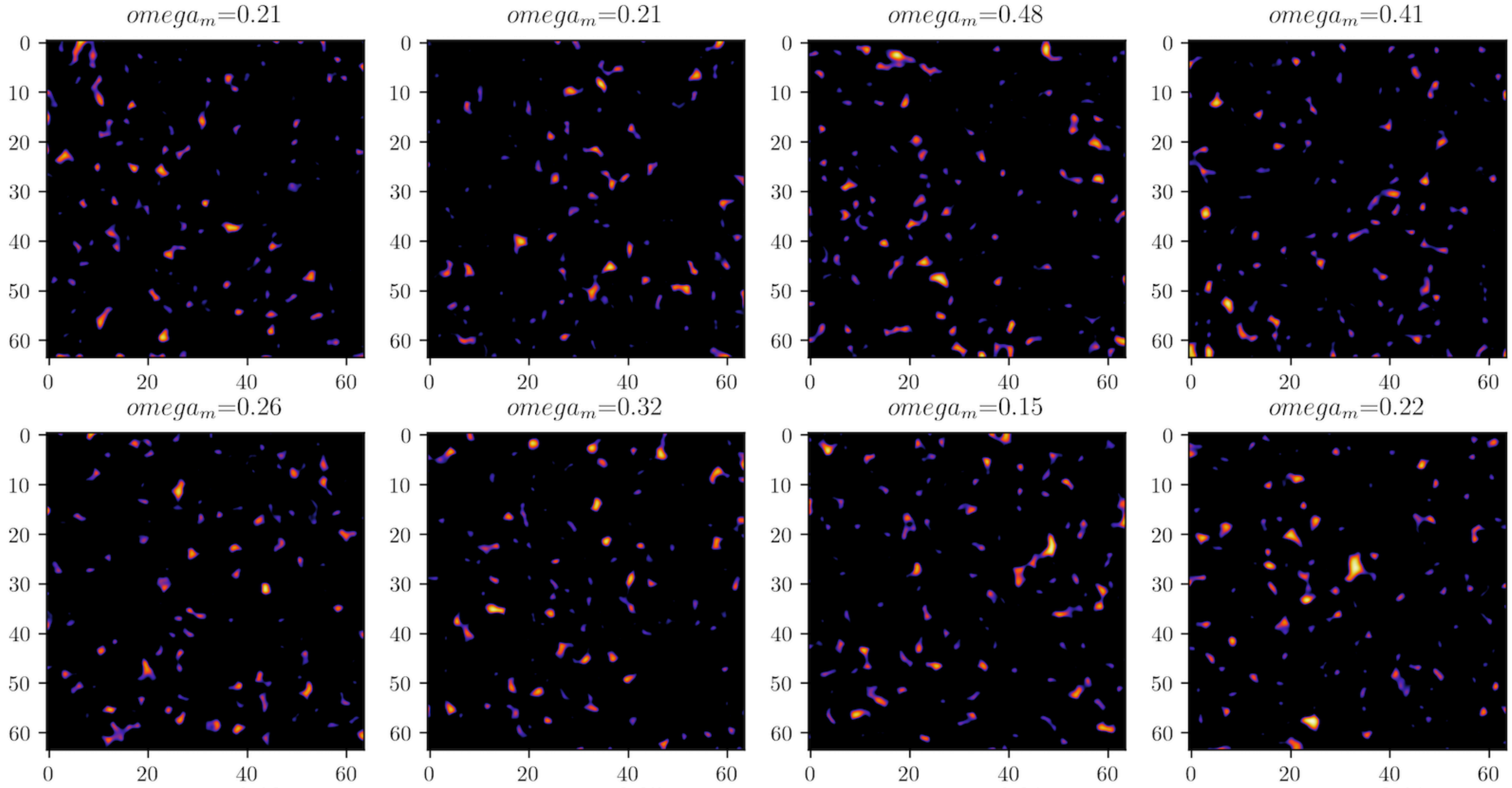
$$P(k) = P_0 k^{-2}$$

Result

$$n(N) \propto N^{-2}$$

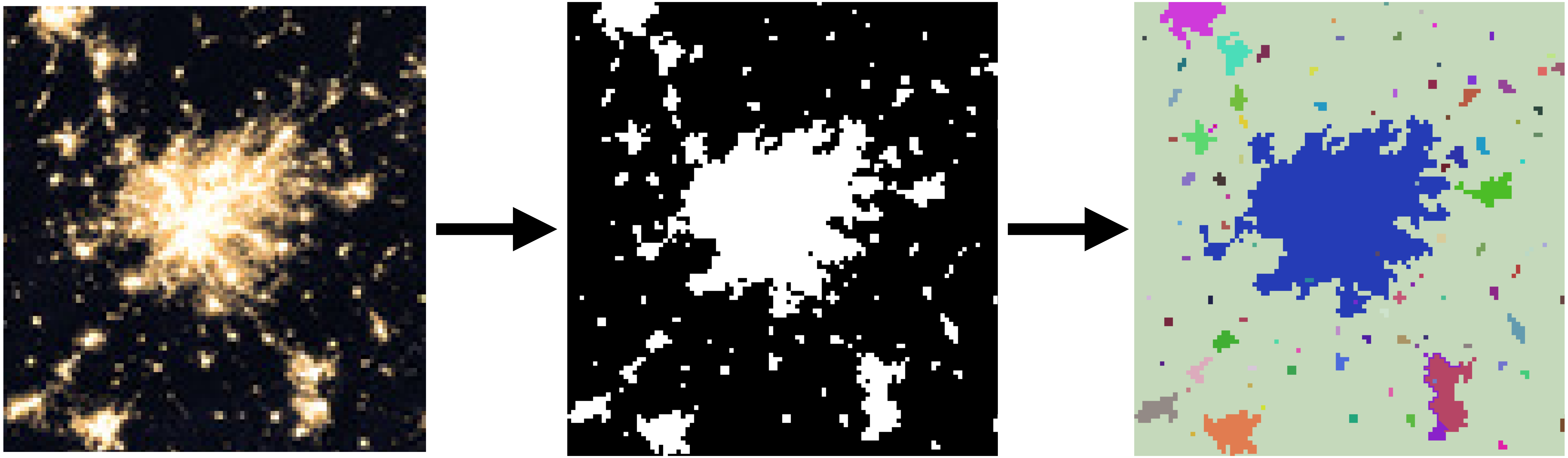
Code

Generating population density fields from an n-body simulator



Code

Analyzing city maps using easily available computer vision tools



Estimating population and “cities”, using light as a surrogate for population density

Conclusions

- Zipf's Law can be derived from population density as the fundamental unit, instead of cities
- This formulation can also be used for other systems (eg social networks[5])
- Simulation code in progress

Aliens?

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Papers that cite

Zipf's law from scale-free geometry

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1 2017Entrp..19..299L 2017/06

Critical Behavior in Physics and Probabilistic Formal Languages

Lin, Henry; Tegmark, Max

2 2016AsBio..16..418L 2016/06

Interstellar Travel and Galactic Colonization: Insights from Percolation Theory and the Yule Process

Lingam, Manasvi

3 2015ApJ...810L...3L 2015/09

Statistical Signatures of Panspermia in Exoplanet Surveys

Lin, Henry W.; Loeb, Abraham

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Papers that cite

Interstellar Travel and Galactic Colonization: Insights from Percolation Theory and the Yule Process

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1 2020AcAau.168..146L 2020/03
Electric sails are potentially more effective than light sails near most stars
Lingam, Manasvi; Loeb, Abraham

2 2020AJ....159...85H 2020/03
Interstellar Communication Network. I. Overview and Assumptions
Hippke, Michael

3 2019IJAsB..18..393H 2019/10
Spaceflight from Super-Earths is difficult
Hippke, Michael

4 2019IJAsB..18..112L 2019/04
Subsurface exolife
Lingam, Manasvi; Loeb, Abraham

5 2019AsBio..19...28L 2019/01
Relative Likelihood of Success in the Search for Primitive versus Intelligent Extraterrestrial Life
Lingam, Manasvi; Loeb, Abraham

6 2018JApA...39...73H 2018/12
Interstellar communication: The colors of optical SETI
Hippke, Michael

7 2018IJAsB..17..116L 2018/04
Physical constraints on the likelihood of life on exoplanets
Lingam, Manasvi; Loeb, Abraham

8 2017PNAS..114.6689L 2017/06
Enhanced interplanetary panspermia in the TRAPPIST-1 system
Lingam, Manasvi; Loeb, Abraham

9 2017ApJ...837L..23L 2017/03
Fast Radio Bursts from Extragalactic Light Sails
Lingam, Manasvi; Loeb, Abraham

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Papers that cite

Statistical Signatures of Panspermia in Exoplanet Surveys

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1	2019AJ....158..117C	2019/09	**The Fermi Paradox and the Aurora Effect: Exo-civilization Settlement, Expansion, and Steady States** Carroll-Nellenback, Jonathan; Frank, Adam; Wright, Jason *and 1 more*
2	2019IJAsB..18..112L	2019/04	**Subsurface exolife** Lingam, Manasvi; Loeb, Abraham
3	2018ApJ...868L..12G	2018/11	**Galactic Panspermia** Ginsburg, Idan; Lingam, Manasvi; Loeb, Abraham
4	2018AsBio..18.1106V	2018/09	**Dynamical and Biological Panspermia Constraints Within Multiplanet Exosystems** Veras, Dimitri; Armstrong, David J.; Blake, James A. *and 3 more*
5	2018exha.book.....P	2018/08	**The Exoplanet Handbook** Perryman, Michael
6	2018ApJ...855L...1C	2018/03	**Habitable Evaporated Cores and the Occurrence of Panspermia Near the Galactic Center** Chen, Howard; Forbes, John C.; Loeb, Abraham
7	2017PNAS..114.6689L	2017/06	**Enhanced interplanetary panspermia in the TRAPPIST-1 system** Lingam, Manasvi; Loeb, Abraham
8	2016JCAP...08..040L	2016/08	**Relative likelihood for life as a function of cosmic time** Loeb, Abraham; Batista, Rafael A.; Sloan, David
9	2016AsBio..16..418L	2016/06	**Interstellar Travel and Galactic Colonization: Insights from Percolation Theory and the Yule Process** Lingam, Manasvi
10	2016MNRAS.455.2792L	2016/01	**Analytical approaches to modelling panspermia - beyond the mean-field paradigm** Lingam, Manasvi

Panspermia

Hypothesis that life exists throughout the Universe and is distributed by various phenomena

Some results

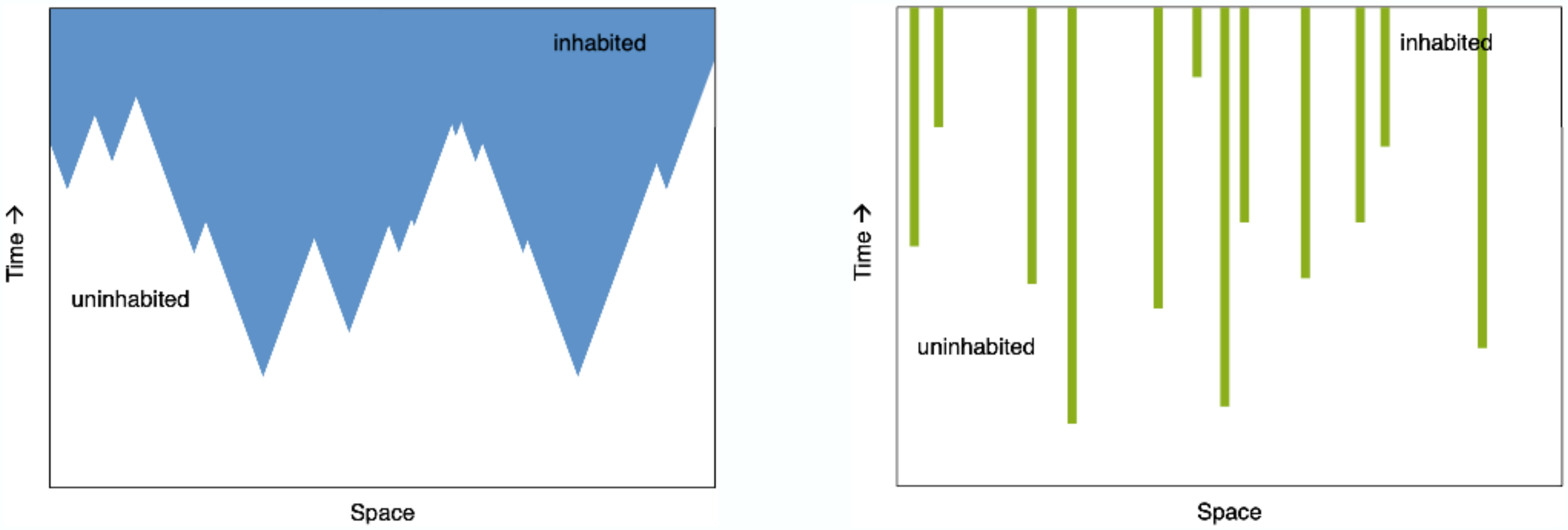

Fig. 1.— Schematic diagrams of the topology of the bio-inhabited planets within the galaxy for the panspermia case (left) and no panspermia case (right). In the panspermia case, once life appears it begins to percolate, forming a cluster that grows with time. Life can ocassionally spontaneously arise after the first bio-event, forming clusters that are smaller than more mature clusters. (The limiting case where life spontaneously arises once and then spreads to the rest of the galaxy would correspond to a single blue triangle. In the "sudden" scenario, all triangles start at the same cosmic time and are thus the same size.) As time progresses, the clusters eventually overlap and the galaxy's end state is dominated by life. In the no panspermia scenario, life cannot spread: there is no phase transition, but a very gradual saturation of all habitable planets with life. Observations of nearby habitable exoplanets could statistically determine whether panspemia is highly efficient (left), inefficient (right), or in some intermediate regime.

Some results

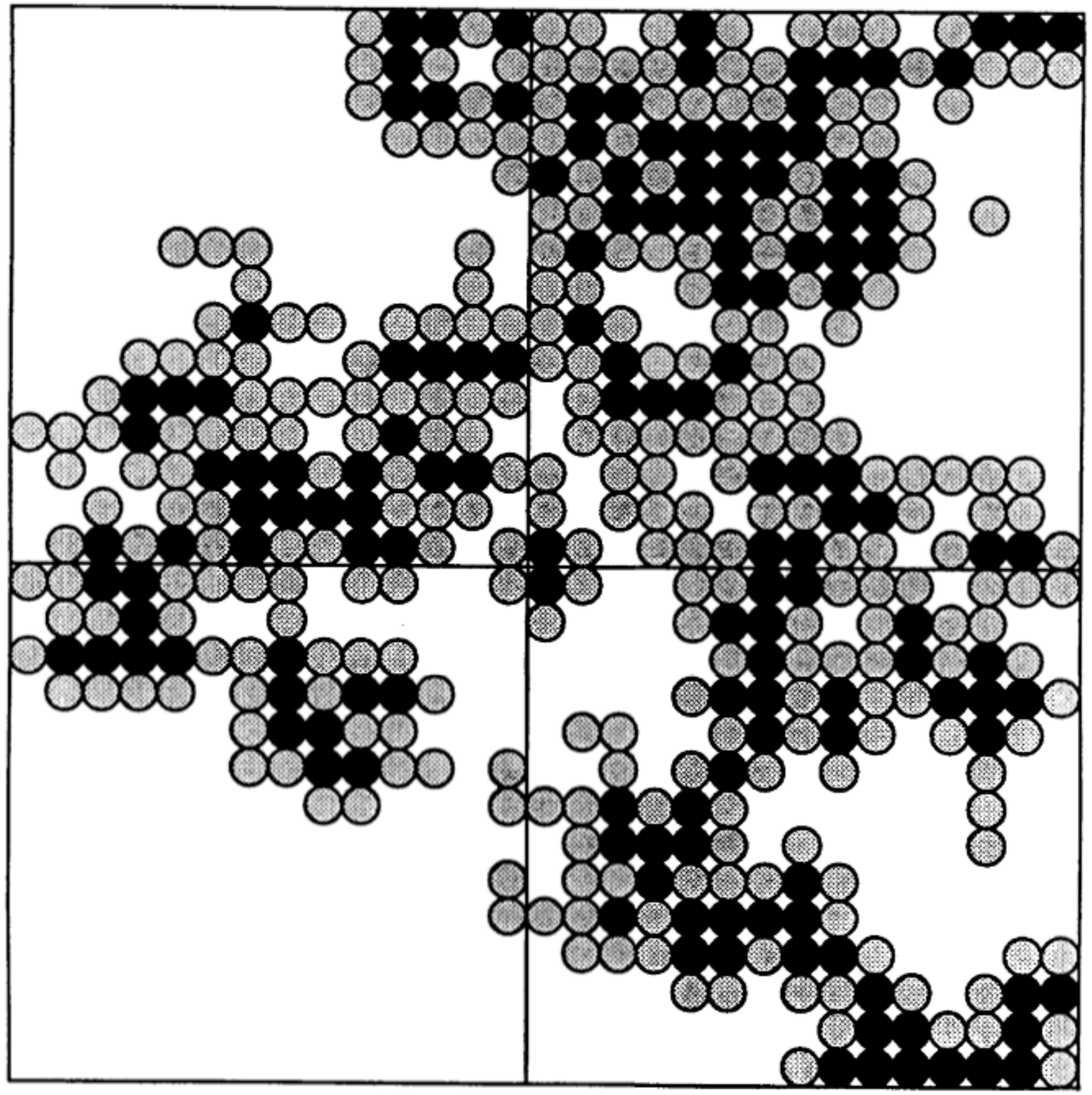

Figure 1. A slice from a percolation simulation on a simple cubic lattice in three dimensions. Here N=6 and P=1/3. Filled circles denote "colonizing" sites, open circles "non-colonizing" sites, and the absence of circles represents sites not visited. The irregular shape of the boundary and large voids in the percolation structure are clearly visible.

- for $p < p_c$, small and isolated clusters are scattered throughout the lattice.
- for $p > p_c$, a giant cluster emerges that spans the entire lattice.

Questions, Comments, Concerns?