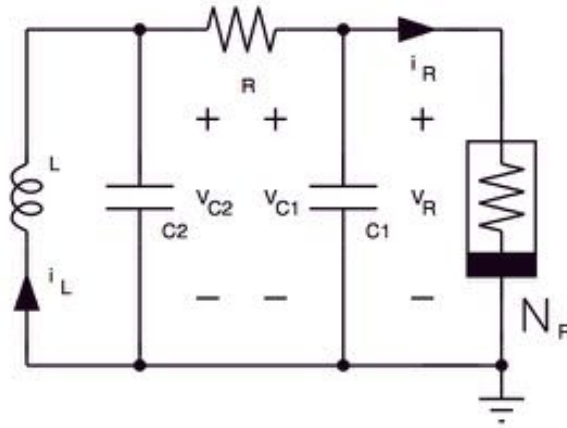


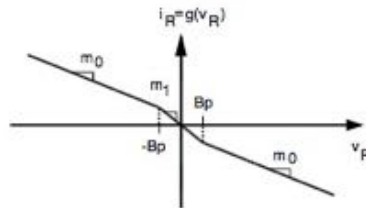
# Chua's Circuit

## A chaotic analog computer

Chua's circuit is a simple circuit that exhibits chaotic behavior. This means that it produces an oscillating waveform that never repeats. The ideal Chua's circuit consists of an inductor, capacitors, resistors and Chua's diode. The inductor  $L$ , the resistor  $R$  and capacitors  $C_1$  and  $C_2$  make up a conventional oscillation circuit. Chua's diode is a nonlinear active resistor whose behavior is described by 3 piecewise linear equations. It is not available as a separate component and has to be realized using off the shelf parts. The following figure shows ideal Chua's circuit[1].



Here,  $L$  is a linear inductor,  $C_2$  and  $C_1$  are capacitors,  $R$  is a resistor and  $N_R$  is Chua's diode. Chua's diode has the following v-i characteristic curve  $g(v)$ [1]:

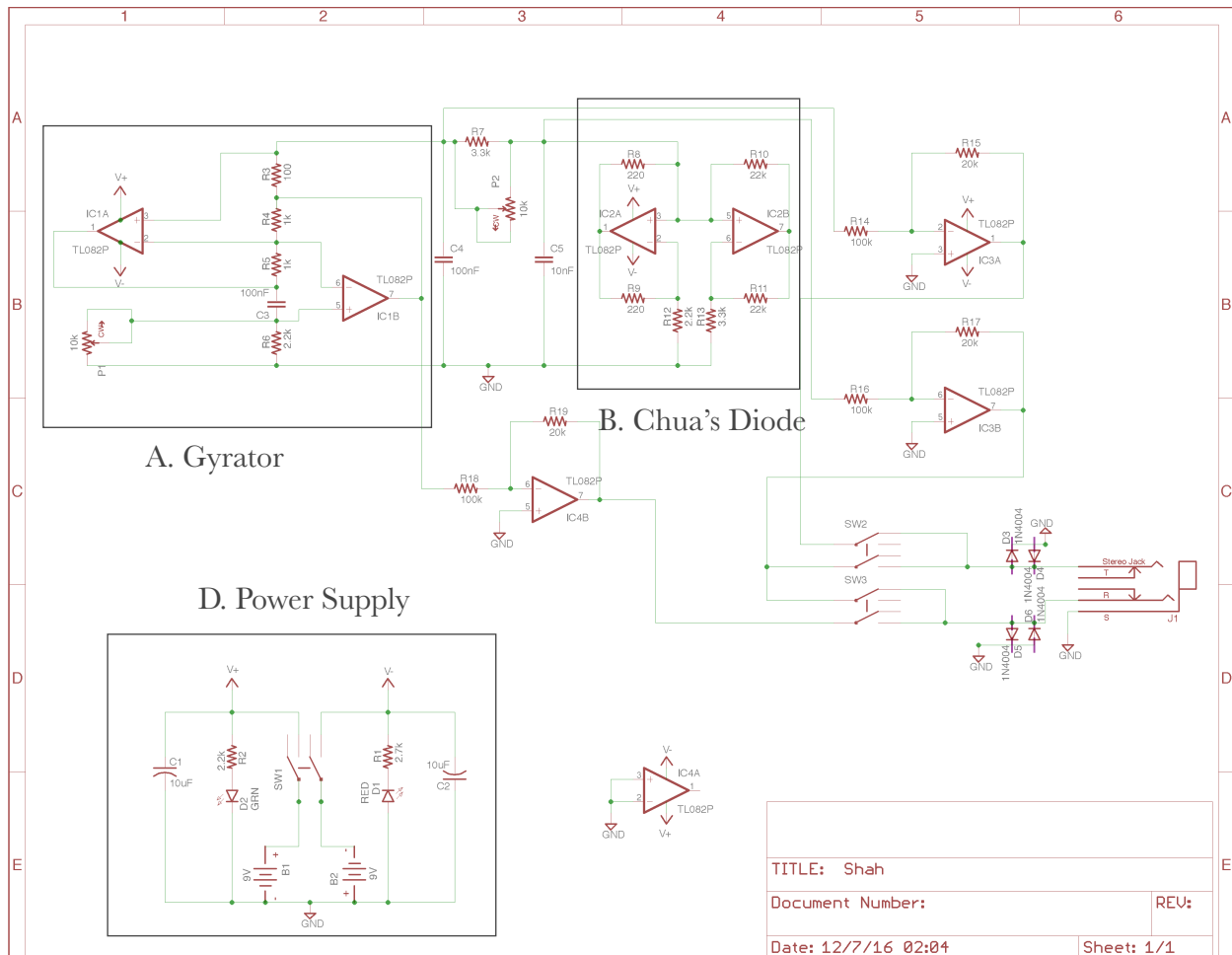


The outer regions have slope  $m_0$ , while the inner region has slope  $m_1$ . Here,  $V_{C_2}$ ,  $V_{C_1}$  and  $i_L$  vary according to the following system of equations:

$$\begin{aligned} C_1 \frac{dv_{C_1}}{dt} &= G(v_{C_2} - v_{C_1}) - g(v_{C_1}) \\ C_2 \frac{dv_{C_2}}{dt} &= G(v_{C_1} - v_{C_2}) + i_L \\ L \frac{di_L}{dt} &= -v_{C_2} \end{aligned}$$

## Implementation

My circuit consists of four parts:



Schematic for my Chua's circuit

### A) The Gyrator:

As this is a chaotic circuit, the stability of its components is extremely important. We also need to be able to manipulate the inductance easily. These problems are solved by using a Gyrator instead of a physical inductor. A Gyrator is a circuit made up of Op-Amps and different impedances (resistors/capacitors). Here, we use a Gyrator consisting of Op-Amps IC1A and IC1B, resistors  $R_6 || P_1$ ,  $R_5$ ,  $R_4$  &  $R_3$ , and capacitor  $C_3$ . It simulates a perfect inductor. The inductance is given by:

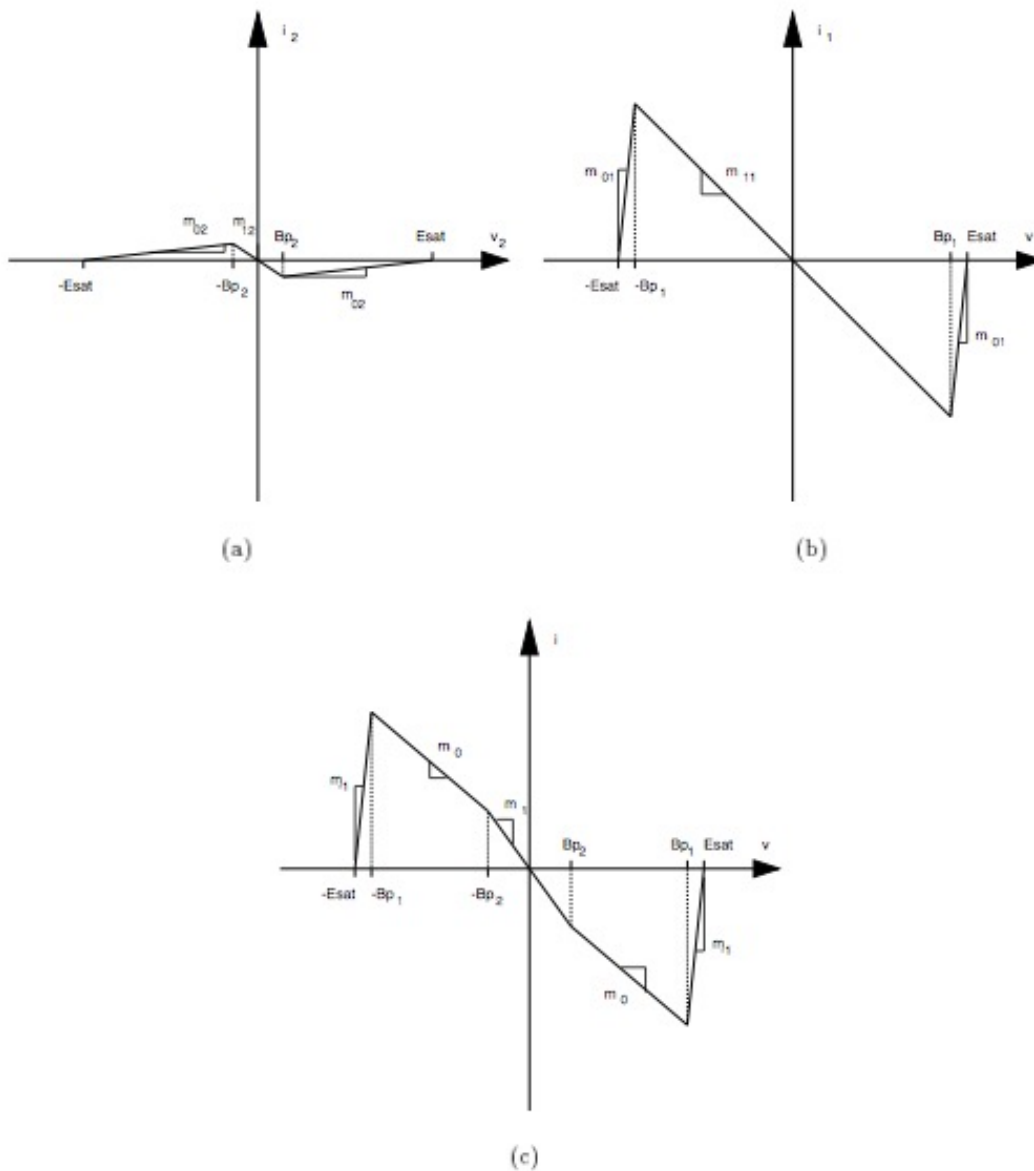
$$L = (R_6 || P_1)R_3R_4C_3 / R_5$$

Where  $P_1$  is a potentiometer. Thus, by varying the resistance of the potentiometer, we can vary the inductance of the gyrator.

The other variable parameter is  $P_2$ . Varying it changes the  $R$  of our RLC circuit.

B) Chua's Diode:

The Chua's diode implementation is based on "Robust Op Amp Realization of Chua's Circuit" by Professor Kennedy (of University College Dublin, not GaTech)[1]. Here, we use two Op Amps IC2A and IC2B as Negative Impedance Convertors., operating in both their linear and nonlinear regions. We use different resistors (3 per Op Amp) to impart different negative resistances on the op amps. By combining them in parallel, we get a piecewise linear function similar to ideal Chua's diode.



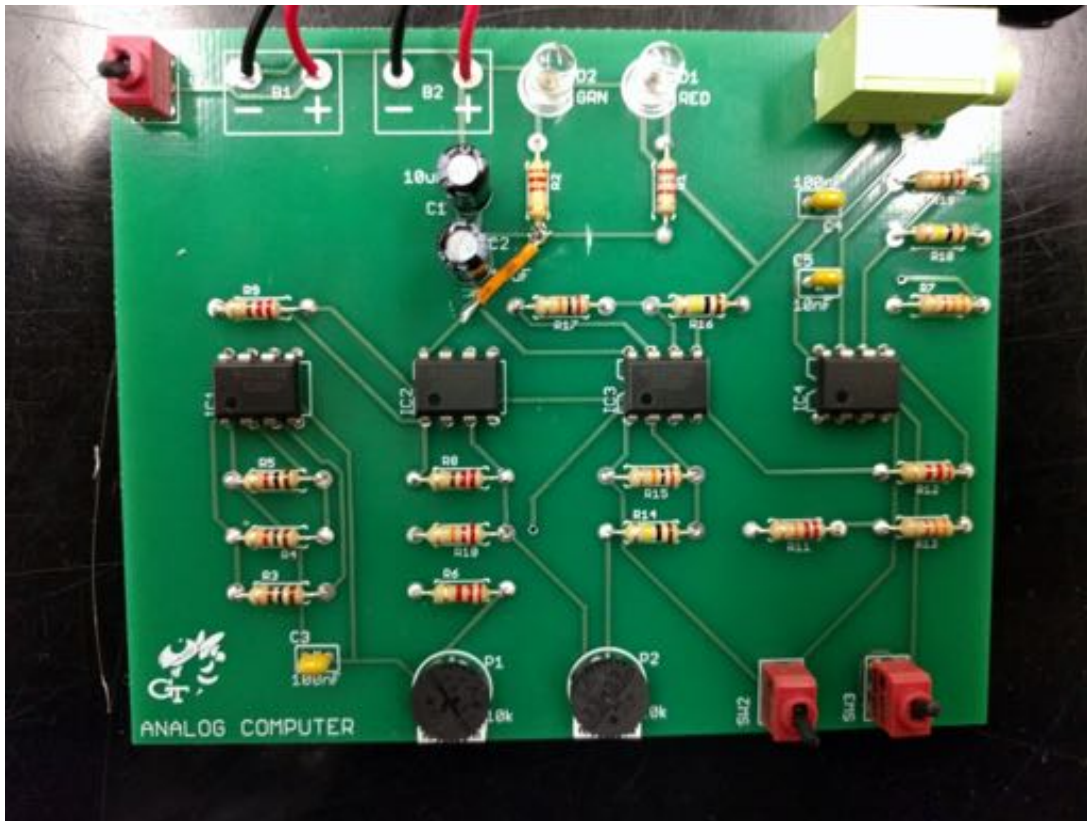
Here, (a) is representative of IC2A's v-i curve, (b) is representative of IC2B's v-i curve, (c) is the v-i curve of IC2A | IC2B. Notice that it resembles the v-i curve of Chua's diode we saw earlier.[1]

C) Output buffer:

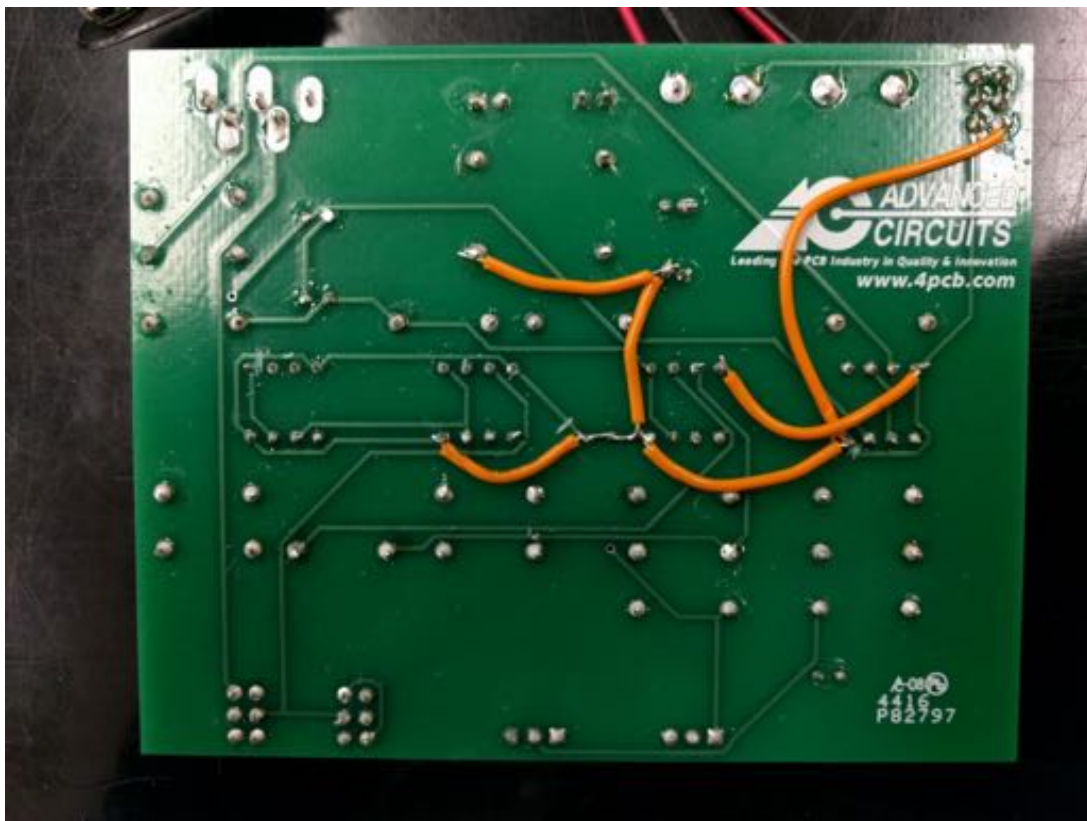
Chua's circuit has 3 outputs: The current passing through the inductor (we use voltage across inductor here) between R3 and R4, and the voltages across both the central capacitors, between R7 and C4, and R7 and C3 respectively. The range of the output can go up to 18V. While this works fine with Oscilloscopes, we need to decrease the range to connect it to a computer through stereo jacks. Here, we use 3 op amps as inverting amplifiers to reduce the range. The transfer function across each of the op amp is  $-1/5$ . The additional diodes clip the range to  $\pm V_{pn}$ . At any given time, we can study two outputs, depending on the configuration of switches S2 and S3.

D) Power Supply: Two 9 V batteries.  $V+ = 9V$  and  $V- = -9V$ . It is controlled by switch S1. Red and Green LEDs show the status of the  $V+$  and  $V-$  power sources respectively.

Photographs of my circuit



Component Side



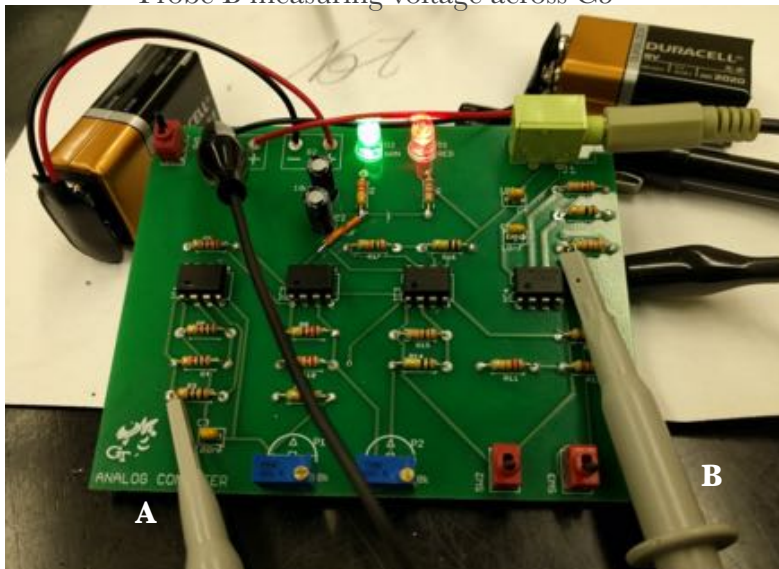
Solder side

## Instructions

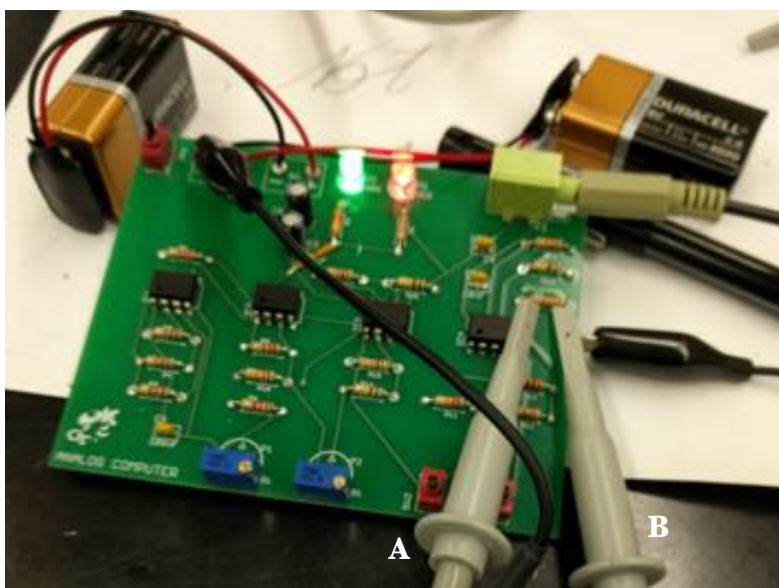
For best plots, use an analog oscilloscope for this. Connect the oscilloscope probes to the outputs on the circuit directly as illustrated below, or to the output from stereo jack. Make sure that you ground the probes. Switch on the circuit and vary the values of the potentiometers. Plot the X-Y graph on oscilloscope. As you vary the potentiometers, you will see different orbits and attractors.

On the circuit directly, you can connect the probes in the following configurations:

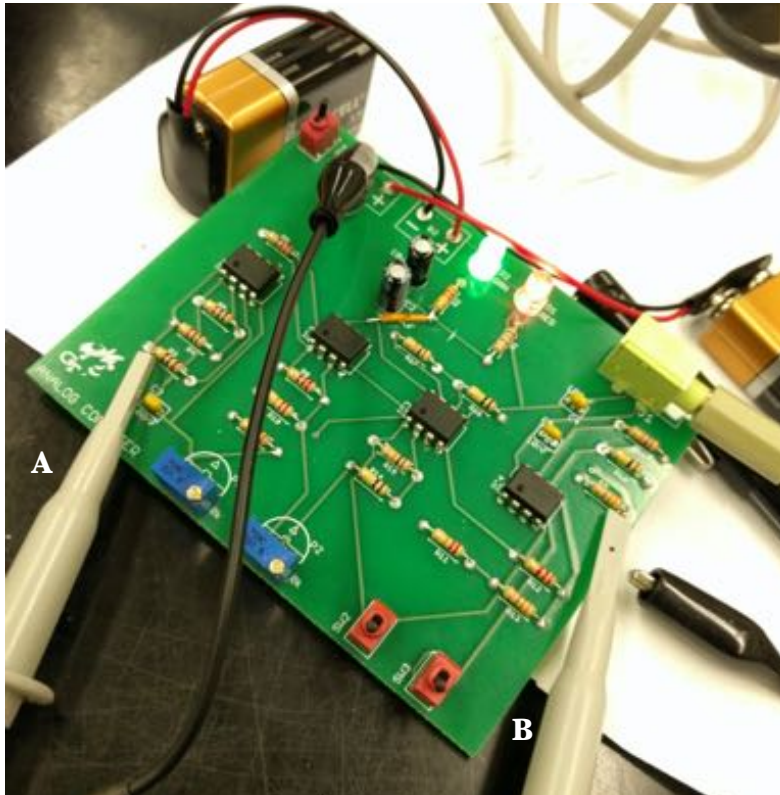
- 1) Probe A measuring voltage across inductor (gyrator),  
Probe B measuring voltage across C5



- 2) Probe A measuring voltage across C4,  
Probe B measuring voltage across C5

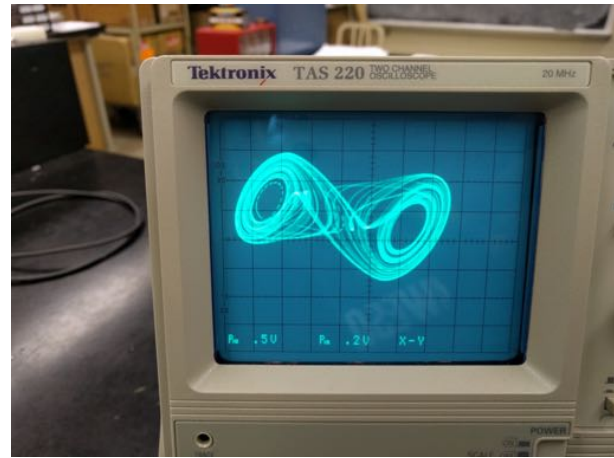
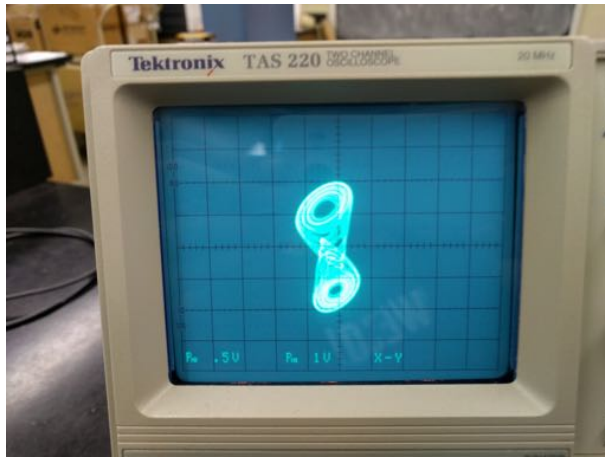


3) Probe A measuring voltage across inductor(gyrator),  
Probe B measuring voltage across C5

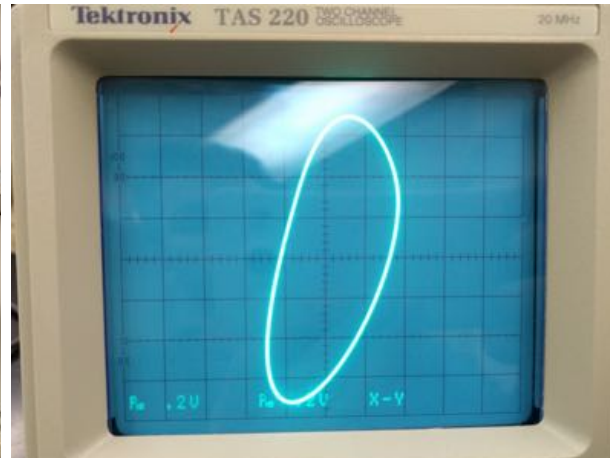
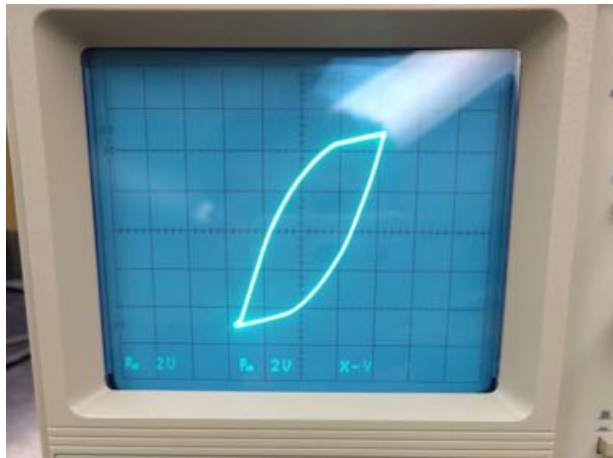


## Oscilloscope Graphs

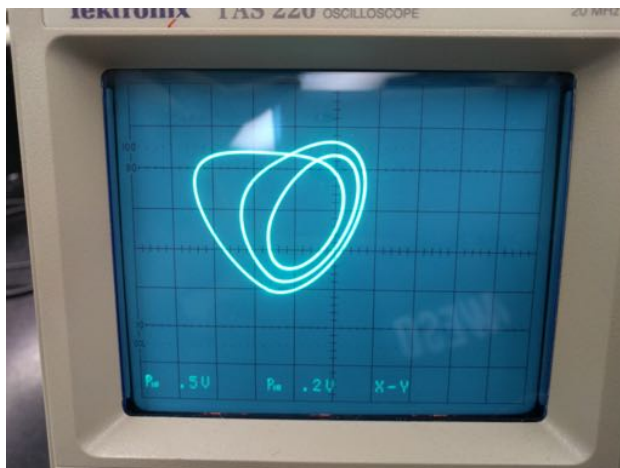
Chua Attractors from configs 3 and 1 respectively:



Limit cycles in configuration 2:



Some stable configuration 2 orbits:

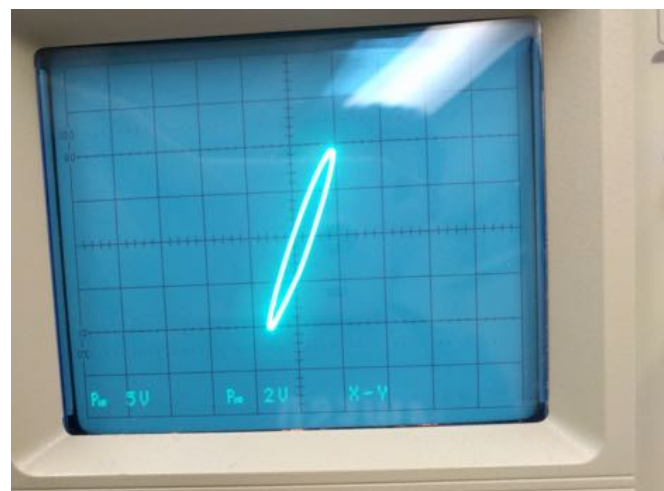
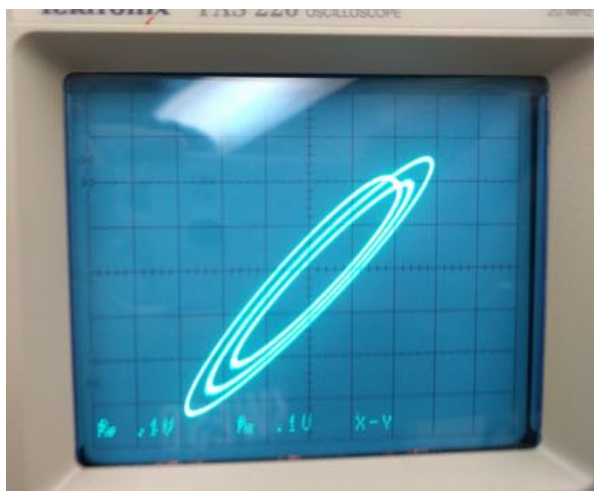




Beginning of chaos in configuration 1: Rössler attractor



Configuration 3 stable orbits:



#### Source

[1] Kennedy, Michael Peter. "Robust Op Amp Realization of Chua's circuit", Frequenz, vol. 46, o. 3-4, March-April 1002, pp 66-80

[2] <http://www.chuacircuits.com> was also helpful

#### Acknowledgments

I am grateful to Dr. First, Dr. Otte and Scott Centers for helping me build and test the circuit.